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BRIEF COMMUNICATION

LARGE-AMPLITUDE KELVIN–HELMHOLTZ WAVES IN GAS–LIQUID FLOWS

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INTRODUCTION

Linear Kelvin–Helmholtz instability is considered a mechanism of central importance in horizontal gas–liquid flows (e.g. Kordyban & Ranov 1970; Andritsos & Hanratty 1987). Nonlinear corrections, however, have received relatively little attention and their significance is not clear. The present work considers the existence and properties of progressive, large-amplitude waves of permanent form on the interface between a gas and a liquid layer of finite depth. The gas shear is modeled by introducing a velocity discontinuity U across the interface. In this inviscid approach the waves are functions of four parameters: the wave amplitude a, the ratio of fluid densities r, the relative velocity of the fluids U and the liquid depth. Fundamental theoretical results in the literature (Bontozoglou & Hanratty 1988) are summarized and subsequently used to infer the nonlinear evolution of interfacial waves. The predictions are shown to compare favorably (at least qualitatively) with the experimental observations.

THEORETICAL RESULTS AND COMPARISONS

Progressive waves of permanent form at the interface between two fluids in relative motion are considered. The fluids are assumed inviscid and the flow, irrotational. The two streams have densities ρ_G and ρ_L, uniform depths h_G and h_L and move co-currently with uniform velocities U_G and U_L. The interface is covered with two-dimensional, periodic waves of amplitude a and wavelength L (wavenumber $k = 2\pi/L$) which propagate with phase speed C in the direction of the flow. It should be noted that, although the next few formulas include all the above parameters, only the limit $h_G \to \infty$ is considered in the present work.

The assumption of waves with infinitesimally small amplitude (linearization) leads to the well-known Kelvin–Helmholtz instability, which may be interpreted as the nonexistence of steady, linear waves of a given wavelength when the current velocity $U = (U_G - U_L)$ is larger than a critical U_{cl} (the subscript stands for critical linear). The value of U_{cl} is given (Milne-Thomson 1968) by the expression

$$U_{cl}^2 = \frac{g}{k}\left(\frac{1-r+\kappa}{r}\right)(\tanh kh_G + r\tanh kh_L), \tag{1}$$

where r is the ratio of densities ρ_G/ρ_L and κ is the ratio of the surface tension forces to gravity forces, $k^2\sigma/\rho_L g$.

Perturbation expansions in the wave amplitude a were used to extend the validity of the above results. The dispersion relation for weakly nonlinear waves at the interface between fluids of arbitrary depth was thus found (Bontozoglou & Hanratty 1988) to be

$$\frac{1}{\tanh kh_L}C^2 + r\frac{1}{\tanh kh_G}(U-C)^2 = \frac{g}{k}(1-r)[1 + a^2 f(k, U, h_L, h_G)], \tag{2}$$

where the expression for f is contained in the original publication. Waves are considered sufficiently long for the effect of surface tension to be neglected.

From the form of the dispersion relation for finite-amplitude waves, it can be seen that there will again be a critical current U_c beyond which steady solutions no longer exist. This limiting behavior is referred to as the dynamical limit, to distinguish from the limit of very high waves due to the occurrence of geometrical singularities and breaking (geometrical limit). Saffman & Yuen (1982) calculated U_c, both analytically (second-order approximation) and numerically, for unbounded fluids. They were the first to note that the critical current velocity increases for increasing wave amplitude a, a result that can be viewed as a stabilization of parallel flows by waves. Thus, for a given value of $U > U_{cl}$, steady interfacial configurations exist on unbounded fluids only if there are waves with heights greater than some minimum.

To extend the dynamical limit results for liquid films of finite depth ($h_G \to \infty$), the value of the critical current velocity U_c, correct to second order in the amplitude a, is obtained by equating the two roots in [2]. For the case of a liquid film of arbitrary depth, the dependence of U_c on the amplitude a of the waves takes the form

$$\left(\frac{U_c}{U_{cl}}\right)^2 = 1 + fk^2a^2. \qquad [3]$$

When the term $[(U_c/U_{cl})^2 - 1]$ is plotted vs k^2a^2, it gives a straight line through the origin with slope f, which varies with liquid depth. For deep liquid ($h_L \to \infty$), the slope takes the limiting value

$$(f)_{deep} = \frac{1 + r^2}{(1 + r)^2}. \qquad [4]$$

Figure 1 shows f, normalized with the deep-fluid slope, as a function of $\exp(-kh_L)$ for three values of the density ratio ($r = 0.1, 0.5, 0.9$). It is interesting to note that for any value of r there are regions where the slope, f, is negative. In these regions an increase in the amplitude a of steady waves gives rise to a decrease in the critical velocity U_c. This is just the opposite of what is found for unbounded fluids, in that, finite-amplitude waves of a given wavelength now cease to exist at current velocities lower than the critical value predicted from linear theory. It should be noted that the above results were also verified numerically. Therefore, the unexpected effect of wave amplitude on U_c for very thin liquid layers is real and not an artiface resulting from ignoring higher-order terms in the expansion.

It is evident from figure 1 that, for gas–liquid systems ($r < 0.1$), there is not much change to the slope until kh_L becomes very small. Then, f decreases abruptly and attains large negative values. It seems, therefore, that there exist two fundamentally different behaviors, one associated with extremely thin films (or, equivalently, very long waves) and the other with thicker ones. Furthermore, unlike liquid–liquid systems ($r = 0.9$), characterized by a gradual transition with decreasing thickness, gas–liquid flows are predicted to exhibit a shock-like transition which should manifest itself in experimental observations.

The results are summarized in figure 2 which shows the variation of $[(U_c/U_{cl})^2 - 1]$ with wave steepness k^2a^2 for two representative cases, one with positive and one with negative slope. Steady wave solutions exist in the region between the negative y-axis and the dynamical limit line. The solution domain is also bounded to the right by the geometrical limit, which is not shown in the graph. It is evident that, for a positive slope, the restriction imposed by the dynamical limit is a minimum wave steepness when $U > U_{cl}$. With a negative slope there are no steady solutions for $U > U_{cl}$ and the restriction is a maximum steepness for $U < U_{cl}$.

Based on the above findings, it is attempted to infer the evolution of linearly unstable disturbances on highly sheared liquids, as a function of the film thickness. Two different behaviors are predicted and are outlined in the following. Concerning thick films (f, positive), it is recalled that the existence of nonlinear, steady solutions at current velocities above the critical linear is associated with supercritical stability (Drazin & Reid 1984; Miles 1986). This notion implies that the fundamental harmonic remains dominant with increasing current velocity, leading to periodic waves of finite amplitude. The range, however, of the wave steepness is restricted by the dynamical limit to values above some minimum. This restriction can be readily met by a decrease in the wavelength, since it is known that the energy per unit area associated with gravity waves is, to first order, proportional to the square of the amplitude and does not depend on the wavelength (Phillips 1977). The increase in the wave steepness cannot continue indefinitely and an upper bound is set

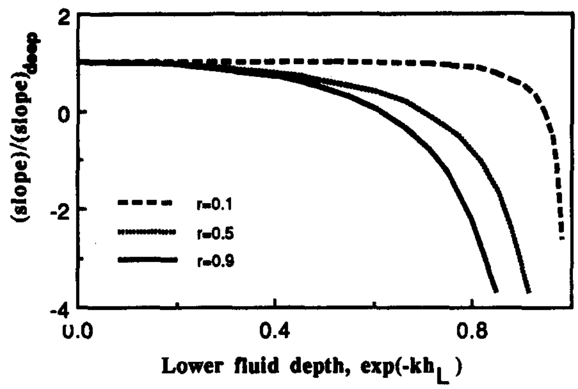

Figure 1. Slope of the $[(U_c/U_{cl})^2 - 1]$ vs a^2 line (normalized with the deep-fluid slope) vs $\exp(-kh_L)$.

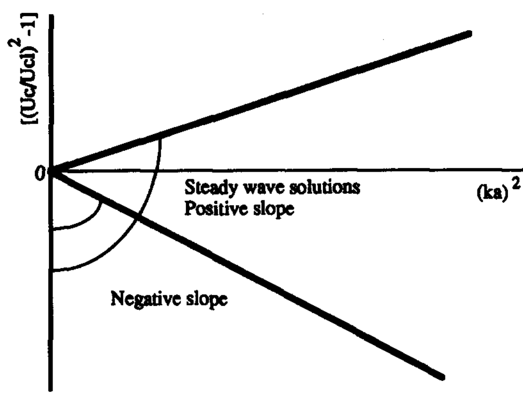

Figure 2. Critical current velocity as a function of wave steepness, from second-order theory.

by the geometrical limit. It has, indeed, been shown elsewhere (Bontozoglou & Hanratty 1989) that this limit correlates the wave steepness actually observed under a wide range of flow conditions.

Extremely thin films (f, negative) which are characterized by the absence of nonlinear, steady solutions for $U > U_{cl}$, exhibit subcritical instability. This implies that above the critical current velocity all superharmonics are simultaneously excited and none of the higher terms may be truncated. The fundamental, linearly unstable wavelength cannot retain its identity and evolve into a nonlinear wave and it is plausibly anticipated that there is a fast transition to a pebbly interface. Thus, the nonexistence of finite-amplitude waves is associated not with a more stable film but with the failure of the system to retain the energy of the instability within a narrow frequency band.

Both of the above predictions have a qualitative resemblance to actual observations in gas–liquid, horizontal flows. Wave properties were measured by Andritsos (1986), who conducted air–water experiments in horizontal pipelines of 2.52 and 9.53 cm dia, with liquid viscosities of 1–80 mPa s. A drastic increase in the wave steepness with a relatively small increase in the air velocity is consistently observed in the data. This behavior occurs at high gas velocities and is caused by a respective decrease in the wavelength. A possible explanation is provided if it is recognized that the range of liquid layer depths is such that the dynamical limit curve always has a positive slope. Increasing the air flow rate results in an increase in the current velocity U $(= U_G - U_L)$. When U exceeds the linear Kelvin–Helmholtz limit U_{cl}, there is a transition from a region where waves of any steepness are dynamically possible to a region where only waves of steepness above some minimum exist. Additional evidence in favor of this argument is provided by figure 3, where the measured wave steepness H/L is plotted vs the dimensionless current velocity ratio $(U_G - U_L)/U_{cl}$ for the relevant data. It is seen that the drastic increase in steepness actually takes place when the current velocity exceeds the linear Kelvin–Helmholtz limit.

A phenomenon, possibly associated with the behavior of the dynamical limit, is observed when roll waves are initiated by air flowing over very thin liquid films. It has been reported (Hanratty

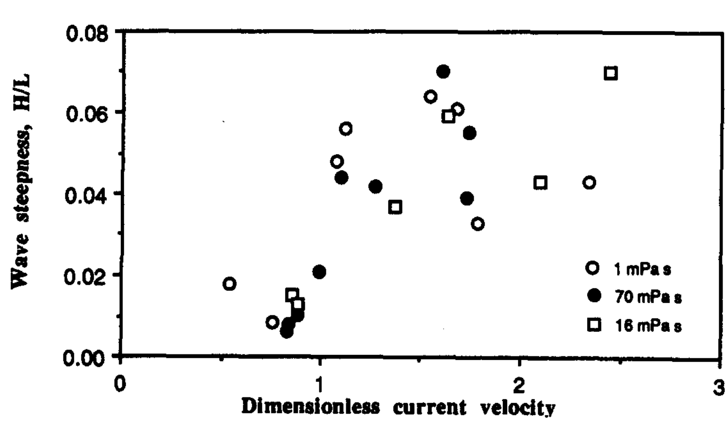

Figure 3. Wave steepness vs the dimensionless current velocity [measurements by Andritsos (1986)].

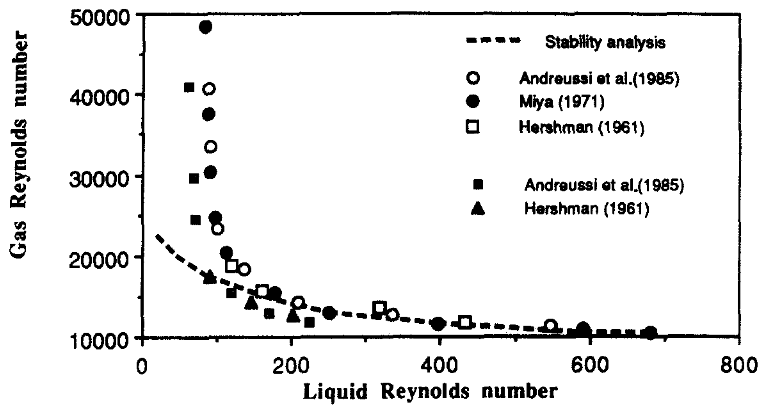

Figure 4. Critical gas velocity for the onset of roll waves as a function of Re_L, from various investigations. Data reproduced from Andreussi *et al.* (1985). The dashed line corresponds to a stability analysis by Hanratty & Hershman (1961).

& Hershman 1961; Miya 1970) that the initiation of roll waves at high gas velocities is insensitive to gas velocity and occurs at an approximately constant liquid flow rate. This is contrary to linear stability analysis (Hanratty & Hershman 1961), which indicates that at sufficiently high gas velocities the film is unstable for all liquid flow rates. Andreussi *et al.* (1985) examined the effect of liquid viscosity on the critical liquid flow rate and proposed an empirical relaxation correction for the stability analysis. A summary of the data, reproduced from this last work, is presented in figure 4.

The previously described theoretical results, concerning the behavior of the dynamical limit for very thin liquid films, could provide an alternative explanation. It could specifically be argued that, when the film thickness decreases below the value which renders the dynamical limit line of negative slope, no roll waves will grow from infinitesimal disturbances even if linear theory predicts instability. Following the evolution of a film moving at constant flow rate (Re_L), as the gas velocity increases, one would expect a progressive thinning of the film owing to the increasing shear. If the film it too thin (in the above sense) by the time the instability gas velocity is reached, no roll waves will be observed. This argument is also in qualitative agreement with the effect of viscosity, since increasing the viscosity results in a thicker liquid film and, therefore, moves the transition to lower liquid flow rates.

CONCLUDING REMARKS

Large-amplitude, progressive waves of permanent form at the interface between two inviscid fluids in relative motion are investigated theoretically. Parameters of key importance in applications, such as the relative velocity of the two fluids and the liquid film thickness, are studied in detail. Attention is focused on nonlinear phenomena, which are examined by extending techniques developed for handling free-surface waves. The state-of-knowledge of the fundamental aspects of the above problem is rather incomplete, at least compared with the more intensively studied case of free-surface waves. Therefore, the theoretical work undertaken led to new results of fundamental interest.

As is well-known, inviscid theory does not point to a single wave for given flow conditions. Rather it predicts that a whole range of waves is acceptable and energy arguments (wind input vs dissipation) should dictate which of the above waves is actually observed. In this sense, its application to the prediction of wave properties in two-phase flow problems is not straightforward. However, as this work has demonstrated to a certain extent, theory and observations are not irrelevant. Inclusion of a gas velocity in the inviscid analysis leads to the dynamical limit for steady, progressive waves, in addition to the geometrical limit which is also known from free-surface waves. The behavior of this limit with changing flow conditions is in qualitative agreement with two-phase flow observations.

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